Synchronized Multimodal Urban Air Mobility

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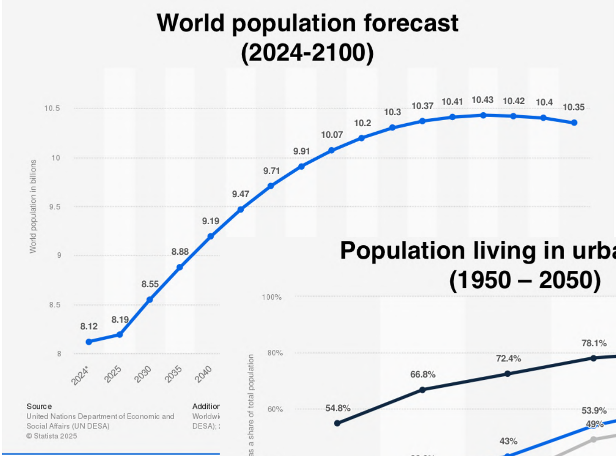
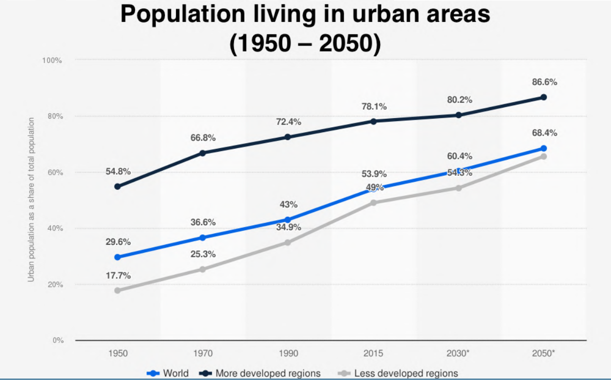
Context – World population and urbanization





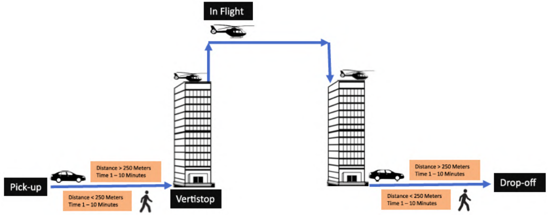
Context – A new mobility paradigm

[EASA (2021)]

Context – eVTOL development and progress

Context – Passenger-centric Urban Air Mobility (UAM)

[Holden & Goel (2016)]



From the literature:

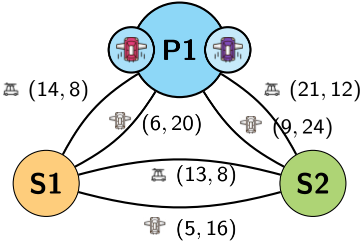
- Vertiplace location [Rajendran & Zack (2019), TRE; Chen et al. (2022), IJOC]
- Scheduling [Shao et al. (2021), TRE; Li et al. (2025), TRC]
- Routing [Bennaceur et al. (2022), TRC; Golden et al. (2025) Annals of OR; Wu et al. (2026), TRE]
- Precedence constraints [Dohn et al. (2011), Net.] or mutual waiting [Luo et al. (2021), NRL]

Research gaps:

- No different types of vertiplaces (landing pads)
- No multimodality
- No battery swapping
- No synchronization
- No potential collisions

Example – Vertiplace-based graph

ID	Origin	Dest.	a	b	q
1	S2	S1	11	18	2
2	S1	P1	22	28	2
3	P1	S2	32	53	2
4	S1	P1	64	76	1
5	S1	P1	64	76	1
6	P1	S1	6	28	2
7	S1	S2	30	40	2
8	S2	S1	50	63	2



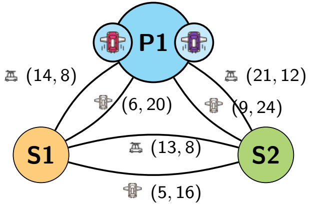
Vertiplace-based graph:

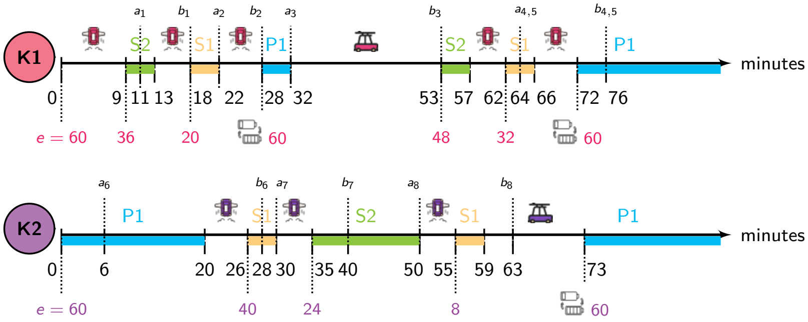
- one vertiport (P1)
- two landing pads and flying cars (K1 and K2; capacity $\overline{q} = 2$)
- two vertistops (S1 and S2)

Example – Timelines

We assume:

- minimum separation time $\tau^{\text{fly}} = 2$
- safety time $\tau^{\text{saf}} = 4$
- total energy $\overline{e} = 60$
- flying threshold $e^F = 18$





Problem definition – Sets and parameters

Multimodal UAM Problem with Synchronization Constraints:

- Planning horizon $[a_0, b_0]$
- $\mathcal{K}$, set of homogeneous eVTOLs:
  - Capacity of $\overline{q}$ passengers
  - $\mathcal{M} := \{\mathsf{F}, \mathsf{G}\}$
  - Energy $[\underline{e}, \overline{e}]$, flying threshold e^{F}
  - Starting and ending their *journeys* (i.e., multi-trips) at some vertiport
- $\mathcal{V} := \mathcal{P} \cup \mathcal{S}$, set of vertiplaces (ports and stops):
  - For each $v \in \mathcal{V}$, number of landing pads ℓ_v ($\ell_v \geq 1$ if $v \in \mathcal{P}$)
  - $\mathcal{H}$, set of all landing pads
- $\mathcal{R}$, set of requests – For each $r \in \mathcal{R}$:
  - origin o_r and destination d_r
  - ready time a_r and due time b_r
  - number of passengers q_r, each with an average weight W

- Minimum separation time τ^{fly} and safety time τ^{saf}
- Payload-dependent energy consumption and battery swapping (in a time negligible w.r.t. τ^{saf})
- Synchronization between pairs of cars using the same pad handled through *sequencing*:

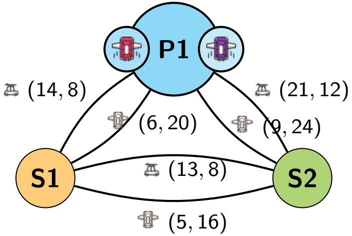
No conflict

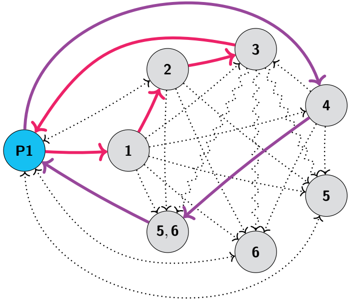
Same vertistop

Same vertiport

- **Goal**: finding a set $\mathcal{J}^*$ of $|\mathcal{K}|$ journeys (i.e., multi-trips) to maximize the number of passengers served

First idea based on groups – Multigraph representation





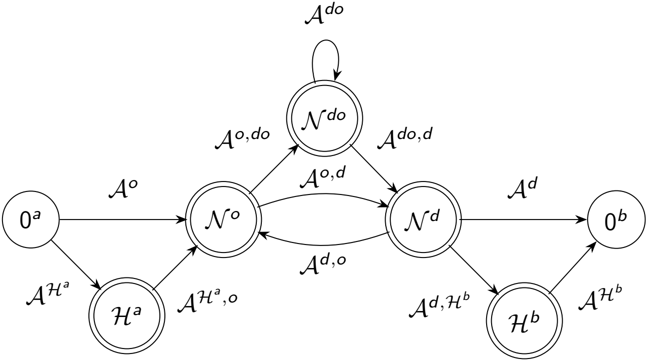
Vertiplace-based graph $(\mathcal{V}, \mathcal{E})$:

- $\mathcal{E} := \{(u, v, m) \mid u, v \in \mathcal{V}, m \in \mathcal{M}\}$
- Travel times t^m_{uv} and energy e^m_{uv}

Group-based multigraph $(\mathcal{G} \cup \mathcal{P}, \mathcal{A}^{\mathcal{G}})$:

- $\mathcal{G}$, set of feasible groups; $\mathcal{A}^{\mathcal{G}} := (g_1, g_2, m) \mid g_1, g_2 \in \mathcal{G} \cup \mathcal{P}, m \in \mathcal{M}\}$
- Travel times $t^m_{g_1 g_2}$ and energy $e^m_{g_1 g_2}$

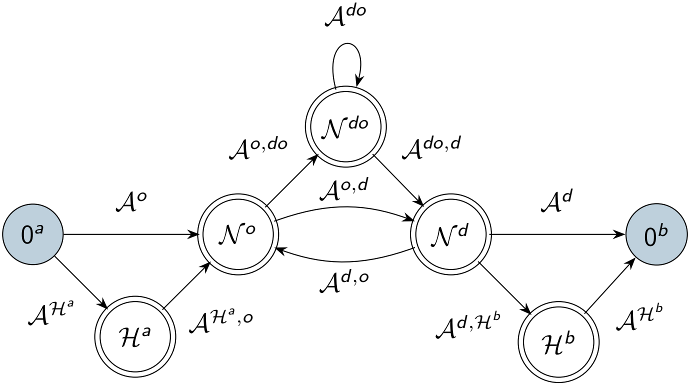
Better idea based on events – Multi-graph representation



Directed event-based multi-graph $(\mathcal{N} \cup \{0^a, 0^b\}, \mathcal{A})$:

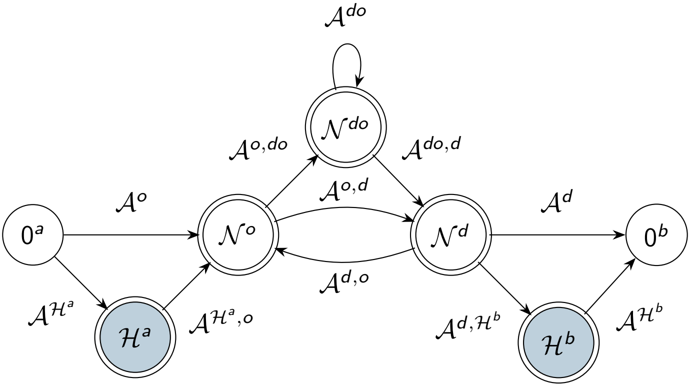
- $\mathcal{N} \cup \{0^a, 0^b\}$, set of possible events; $\mathcal{A}$, set of arcs
- Travel times $t_{ij}^m = t_{\mathcal{V}(i)\mathcal{V}(j)}^m$ and energy $e_{ij}^m = e_{\mathcal{V}(i)\mathcal{V}(j)}^m$, for each $(i, j, m) \in \mathcal{A} \setminus \{\mathcal{A}^o \cup \mathcal{A}^{\mathcal{H}^a} \cup \mathcal{A}^d \cup \mathcal{A}^{H^b}\}$

Better idea based on events – Multi-graph representation



0^a and 0^b: dummy points for the **beginning** a_0 and the **end** b_0

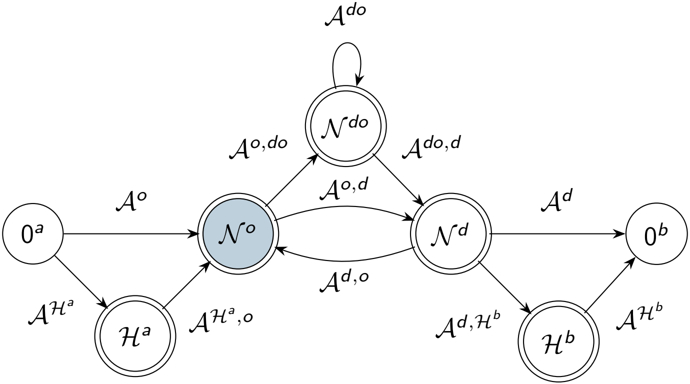
Better idea based on events – Multi-graph representation



$\mathcal{H}^a := \{h \mid p \in \mathcal{P}, h \in \mathcal{H}(p)\}$: **car parking** at a pad at the beginning a_0, from which a car **departs empty**

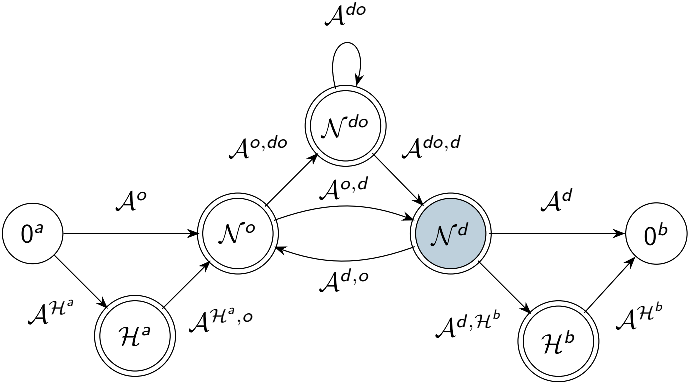
$\mathcal{H}^b$: **car parking** at a pad at the end b_0, **arriving empty**

Better idea based on events – Multi-graph representation



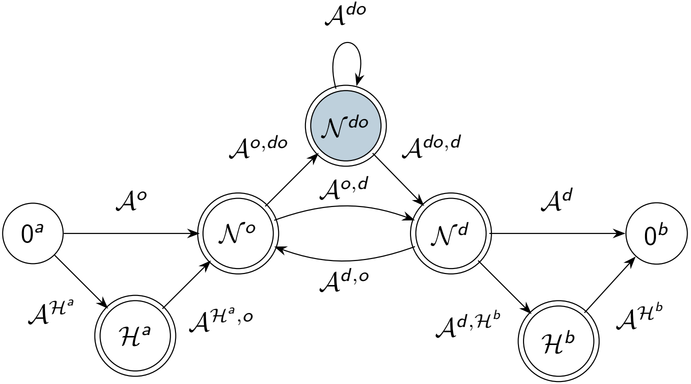
$\mathcal{N}^o := \{n_{o_g} \mid g \in \mathcal{G}\}$: **pickup** of a group g at its vertiplace $o_g \in \mathcal{V}$, at which the car **arrives empty**

Better idea based on events – Multi-graph representation

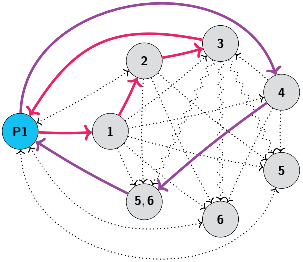


$\mathcal{N}^d := \{n_{d_g} \mid g \in \mathcal{G}\}$: **delivery** of a group g at the vertiplace $d_g \in \mathcal{V}$, from which the car **departs empty** right after

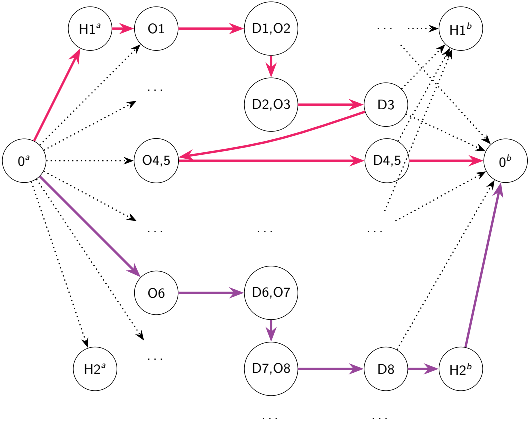
Better idea based on events – Multi-graph representation



$\mathcal{N}^{do} := \{n_{d_{g_1} o_{g_2}} \mid g_1, g_2 \in \mathcal{G}, g_1 \neq g_2, d_{g_1} = o_{g_2}, a_{g_1} \leq b_{g_2}\}$:
delivery of a group g_1 at the vertiplace $d_{g_1} = o_{g_2} \in \mathcal{V}$ and
subsequent pickup of another group g_2



Group-based multigraph
$(\mathcal{G} \cup \mathcal{P}, \mathcal{A}^{\mathcal{G}})$



Event-based multigraph
$(\mathcal{N} \cup \{0^a, 0^b\}, \mathcal{A})$

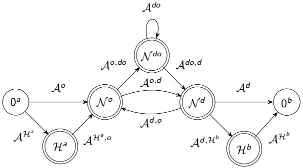
Event-based representation – Conflict set

- $\Phi := \{\mathcal{O}, \mathcal{D}\}$, set of service phases
- $\Gamma := \{\text{arr}, \text{dep}\}$, set of operations
- Given a request $r \in \mathcal{R}$:
  - Origin $o_r = \mathcal{V}(\mathcal{O}_r)$ and destination $d_r = \mathcal{V}(\mathcal{D}_r)$
  - Given $\varphi \in \Phi$ and $\gamma \in \Gamma$, time window $[a_r^{\varphi,\gamma}, b_r^{\varphi,\gamma}]$
- $\mathcal{C} := \{(r, \varphi, s, \phi)\}$, set of potential conflicts between pairs of (request, phase):

Proposition

Given any two pairs (r, φ) and (s, ϕ), with $[a_r^{\varphi}, b_r^{\varphi}]$ and $[a_s^{\phi}, b_s^{\phi}]$, between (r, φ) and (s, ϕ), it exclusively holds one of the following:

1. ***No conflict**: If r or s is not served or $\mathcal{V}(\varphi) \neq \mathcal{V}(\phi)$ or $b_r^{\varphi} < a_s^{\phi}$ or $b_s^{\phi} < a_r^{\varphi}$ or same grouping, then $(r, \varphi, s, \phi) \notin \mathcal{C}$.*
2. ***Handling at the same vertistop**: If $\mathcal{V}(\varphi) = \mathcal{V}(\phi) \in \mathcal{S}$, then either (r, φ) is sequenced before (s, ϕ) or vice versa, and $(r, \varphi, s, \phi) \in \mathcal{C}$.*
3. ***Handling at the same vertiport**: If $\mathcal{V}(\varphi) = \mathcal{V}(\phi) \in \mathcal{P}$, then either (r, φ) is sequenced before (s, ϕ), or vice versa, or (r, φ) and (s, ϕ) overlap each other but at different pads, and $(r, \varphi, s, \phi) \in \mathcal{C}$.*



Decision variables:

- $x_{ij}^{m} \in \{0,1\}$, $(i,j,m) \in \mathcal{A}$, equal to 1 if the arc (i,j,m) is selected
- $\theta_r^{\varphi,\gamma} \in \mathbb{N}$, $r \in \mathcal{R}', \varphi \in \Phi, \gamma \in \Gamma$, the time the operation γ is performed, related to the phase φ of a request r
- $\varepsilon_r^{\varphi} \in \mathbb{R}_+$, $r \in \mathcal{R}', \varphi \in \Phi$, the energy left after reaching a node representing the phase φ of a request r
- $z_{rs}^{\varphi\phi} \in \{0,1\}$, $(r,\varphi,s,\phi) \in \mathcal{C}$, equal to 1 if the potential conflict between (r,φ) and (s,ϕ) is active
- $u_{rs}^{\varphi\phi}$ $(u_{sr}^{\phi\varphi}) \in \{0,1\}$, $(r,\varphi,s,\phi) \in \mathcal{C}$, equal to 1 if (r,φ) is sequenced before (s,ϕ) (if (s,ϕ) is sequenced before (r,φ))
- $y_{rs}^{\varphi\phi} \in \{0,1\}$, $(r,\varphi,s,\phi) \in \mathcal{C}$, equal to 1 if (r,φ) and (s,ϕ) overlaps at different pads of the same vertiplace $\mathcal{V}(\varphi_r)$

Constraints:

- Fleet size, flow conservation, request service, request grouping, temporal conservation, energy conservation, variable domain
- Synchronization – For each $(r, \varphi, s, \phi) \in \mathcal{C}$:

$$u_{rs}^{\varphi\phi} + u_{sr}^{\phi\varphi} + y_{rs}^{\varphi\phi} + \sum_{\substack{(i,j,m)\in \\ \mathcal{A}^{\text{group}}(r,s)}} x_{ij}^{m} = z_{rs}^{\varphi\phi} \qquad \textbf{Conflict handling}$$

$$z_{rs}^{\varphi\phi} \geq \sum_{\substack{(i,j,m)\in\mathcal{A}^{\text{out}}(r,\varphi): \\ s\notin\mathcal{G}(i)}} x_{ij}^{m} + \sum_{\substack{(i,j,m)\in\mathcal{A}^{\text{out}}(s,\phi): \\ r\notin\mathcal{G}(i)}} x_{ij}^{m} - 1 \qquad \textbf{Active conflict}$$

$$\theta_{r}^{\varphi,\text{dep}} + \tau^{\text{fly}} \sum_{\substack{(i,k,F)\in \\ \mathcal{A}^{\text{out}}(r,\varphi)}} x_{ik}^{\text{F}} \leq \left(\theta_{s}^{\phi,\text{arr}} - \tau^{\text{fly}} \sum_{\substack{(k,j,F)\in \\ \mathcal{A}^{\text{in}}(s,\phi)}} x_{kj}^{\text{F}} \right) + M(1 - u_{rs}^{\varphi\phi}) \qquad r \prec s$$

$$\theta_{s}^{\phi,\text{dep}} + \tau^{\text{fly}} \sum_{\substack{(k,j,F)\in \\ \mathcal{A}^{\text{out}}(s,\phi)}} x_{kj}^{\text{F}} \leq \left(\theta_{r}^{\varphi,\text{arr}} - \tau^{\text{fly}} \sum_{\substack{(k,i,F)\in \\ \mathcal{A}^{\text{in}}(r,\varphi)}} x_{ki}^{\text{F}} \right) + M(1 - u_{sr}^{\phi\varphi}) \qquad s \prec r$$

$$y_{rs}^{\varphi\phi} = 0 \quad (r, \varphi, s, \phi) \in \mathcal{C}, \mathcal{V}(\varphi_r) \in \mathcal{S}$$

No simultaneous op. at stops

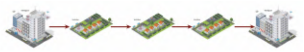
$$\sum_{\substack{(r,\varphi,s,\phi) \\ \in \mathcal{C}}} y_{rs}^{\varphi\phi} + \sum_{\substack{(s,\phi,r,\varphi) \\ \in \mathcal{C}}} y_{sr}^{\phi\varphi} \leq |\mathcal{H}(\mathcal{V}(\varphi_r))| - 1 \qquad r \in \mathcal{R}, \varphi \in \Phi, \mathcal{V}(\varphi_r) \in \mathcal{P}$$

Simultaneous op. at ports

Objective function [Golden et al. (2025), Annals of OR] :

$$\max \quad \sum_{(i,j,m) \in \mathcal{A}^{o,d} \cup \mathcal{A}^{o,do}} q_{\mathcal{G}(i)} x_{ij}^m + \sum_{(i,j,m) \in \mathcal{A}^{do} \cup \mathcal{A}^{do,d}} q_{\mathcal{G}^o(i)} x_{ij}^m$$

Integer relaxations – Trips and journeys



Trip $T :=$ $(p_T^{\mathcal{O}}, \mathcal{G}_T, \mathcal{M}_T, \epsilon_T, \boldsymbol{\tau}_T, p_T^{\mathcal{D}})$:

- Sequence of timed visits between a pair of vertiports
- Visited groups, travel modes, energy consumed, and temporal information
- Enumeration of all feasible trips

Journey $J := \{T_1, T_2, \ldots, T_\rho\}$:

- Sequence of non-overlapping trips
- Enumeration of all feasible journeys

$$
\begin{aligned}
\max \quad & \sum_{J \in \mathcal{J}} q_J \cdot \lambda_J \\
\text{s.t.} \quad & \sum_{J \in \mathcal{J} : r \in \mathcal{R}_J} \lambda_J \leq 1 && r \in \mathcal{R} \\
& \sum_{J \in \mathcal{J}} \lambda_J \leq K \\
& \#\ \text{pads} + \text{temporal constraints} \\
& \text{synchronization constraints} \\
& \lambda_J \in \{0, 1\} && J \in \mathcal{J}
\end{aligned}
$$

Synchronization handled with feasibility cuts – Once computed $\mathcal{J}^*$:

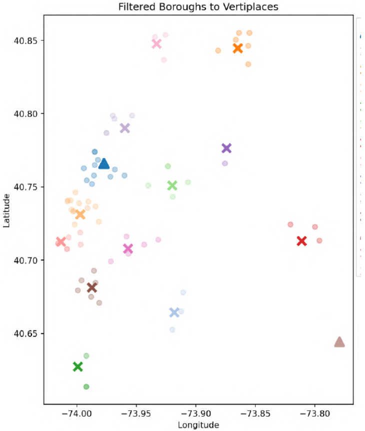
- **Trip-based (weakest)**: $\sum_{J \in \mathcal{J}} \sum_{T \in \mathcal{T}_{\mathcal{J}^*}} \alpha_{TJ} \lambda_J \leq |\mathcal{T}_{\mathcal{J}^*}| - 1$
- **Group-based**: $\sum_{J \in \mathcal{J}} \sum_{g \in \mathcal{G}_{\mathcal{J}^*}} \alpha_{gJ} \lambda_J \leq |\mathcal{G}_{\mathcal{J}^*}| - 1$
- **Request-based (strongest)**: $\sum_{J \in \mathcal{J}} \sum_{\substack{g \in \mathcal{G}_r: \\ r \in \mathcal{R}_{\mathcal{J}^*}}} \alpha_{gJ} \lambda_J \leq |\mathcal{R}_{\mathcal{J}^*}| - 1$

...but not working good so far

Experimental evaluation – Setting

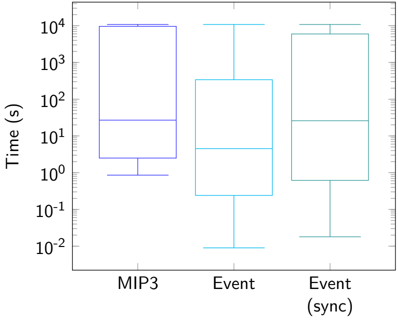
- Implementation in C++
- Solver: CPLEX 22.10
- Two sets from [Golden et al. (2025)]
- One set of realistic instances based on [Holden & Goel (2016), Rajendran & Zack (2019)] and NYC taxi data

Parameter	WRR	GOR-A	GOR-B
Number of instances	180	40	50
$\lvert\mathcal{R}\rvert$	[10, 60] in steps of 10	[5, 200] in steps of 5	[55, 300] in steps of 5
$\lvert\mathcal{V}\rvert$	14	4	8
$\lvert\mathcal{H}\rvert$	4 per vertiport 1 per vertistop	1 per vertiport	1 per vertiport
$\lvert\mathcal{K}\rvert$	4	4	8
$\overline{q}$	3	6	6
Time horizon length (min)	{60, 120}	60	60
Max waiting time (min)	{3, 5, 10, 15, 20}		
Avg TW width (min)	MWT + $t^F_{o_g d_g}$	3	3
τ^{saf} (min)	6	1	1
τ^{fly} (min)	2	–	–
Charge depletion (kW/min)	–	5	5
Payload-dep. cons. (kW)	$\{71, 77, 82, 87\}$	-	-
$\underline{e}$ (kWh)	28 (20% of $\overline{e}$)	0	0
e^F (kWh)	56 (40% of $\overline{e}$)	0	0
$\overline{e}$ (kWh)	140	100	100

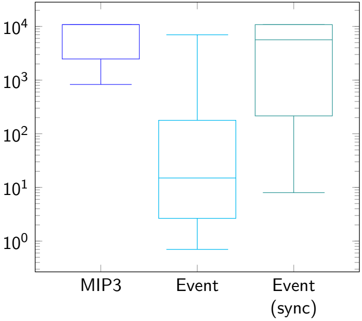


Experimental evaluation – Results over Golden et al.

- Linear recharge/discharge vs battery swapping → Adaptation:
  - τ^{saf} and τ^{fly} set to 0
  - Variables for battery charging time at vertiports and update of energy constraints
- Comparison with MIP3, 3-hour time limit per instance
- No impact of synchronization on optimal solutions



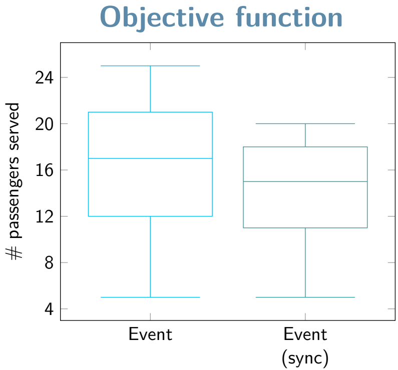
`GOR-A` (5 – 75 requests)

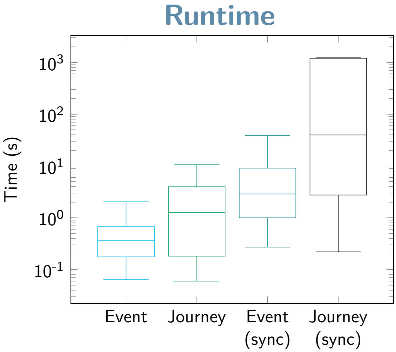


`GOR-B` (55 – 105 requests)

Experimental evaluation – Results over realistic instances

- 1-hour time horizon, 3-min MWT, 20-min time limit





- ↑ MWT, ↑ # passengers served
- ↑ time horizon and ↑ MWT, ↓ size of closed instances (e.g., with 2-hour time horizon, 20-min MWT, and synchronization, up to 20 requests)

Conclusions

Ongoing work:

- Improve the journey-based model and BPC approaches
- Perform sensitivity analyses:
  - Fleet size
  - Number of vertiplaces and landing pads
  - Impact of temporal parameters on synchronization relevance

Future directions:

- Metaheuristics for larger instances
- Stochastic-dynamic setting considering uncertainty (e.g., travel times, energy consumption, request arrival rate)
- Other UAM problems

References

Bennaceur, M., Delmas, R., & Hamadi, Y. (2022). Passenger-centric Urban Air Mobility: Fairness trade-offs and operational efficiency. Transportation Research Part C: Emerging Technologies, 136, 103519.

Chen, L., Wandelt, S., Dai, W., & Sun, X. (2022). Scalable vertiport hub location selection for air taxi operations in a metropolitan region. INFORMS Journal on Computing, 34(2), 834-856.

Dohn, A., Rasmussen, M. S., & Larsen, J. (2011). The vehicle routing problem with time windows and temporal dependencies. Networks 58(4):273–289.

EASA (2021). Study on the societal acceptance of urban air mobility in Europe. Available: `https://www.easa.europa.eu/sites/default/files/dfu/uam-full-report.pdf`.

Golden, B., Oden, E., & Raghavan, S. (2023). The urban air mobility problem. Annals of Operations Research, 1-41.

Holden, J., & Goel, N. (2016). Fast-Forwarding to a Future of On-Demand Urban Air Transportation. Available: `https://evtol.news/__media/PDFs/UberElevateWhitePaperOct2016.pdf`

Li, S., Zhang, T., Xiao, Y., & Li D. (2025). On-demand ridesharing based on dynamic scheduling in urban air mobility. Transportation Research Part C: Emerging Technologies 175:105111.

Luo, Z., Qin, H., Zhu, W., & Lim, A. (2016). Branch-and-price-and-cut for the manpower routing problem with synchronization constraints. Naval Research Logistics (NRL) 63(2):138–171.

Rajendran, S., & Zack, J. (2019). Insights on strategic air taxi network infrastructure locations using an iterative constrained clustering approach. Transportation Research Part E: Logistics and Transportation Review, 128, 470-505.

Shao, Q., Shao, M., & Lu, Y. (2021). Terminal area control rules and evtol adaptive scheduling model for multi-vertiport system in urban air mobility. Transportation Research Part C: Emerging Technologies 132:103385.

Thank you for your attention!

Any question or feedback is welcome