"*A combinatoria of experiences, information, books*": Searching for my own optimal solution

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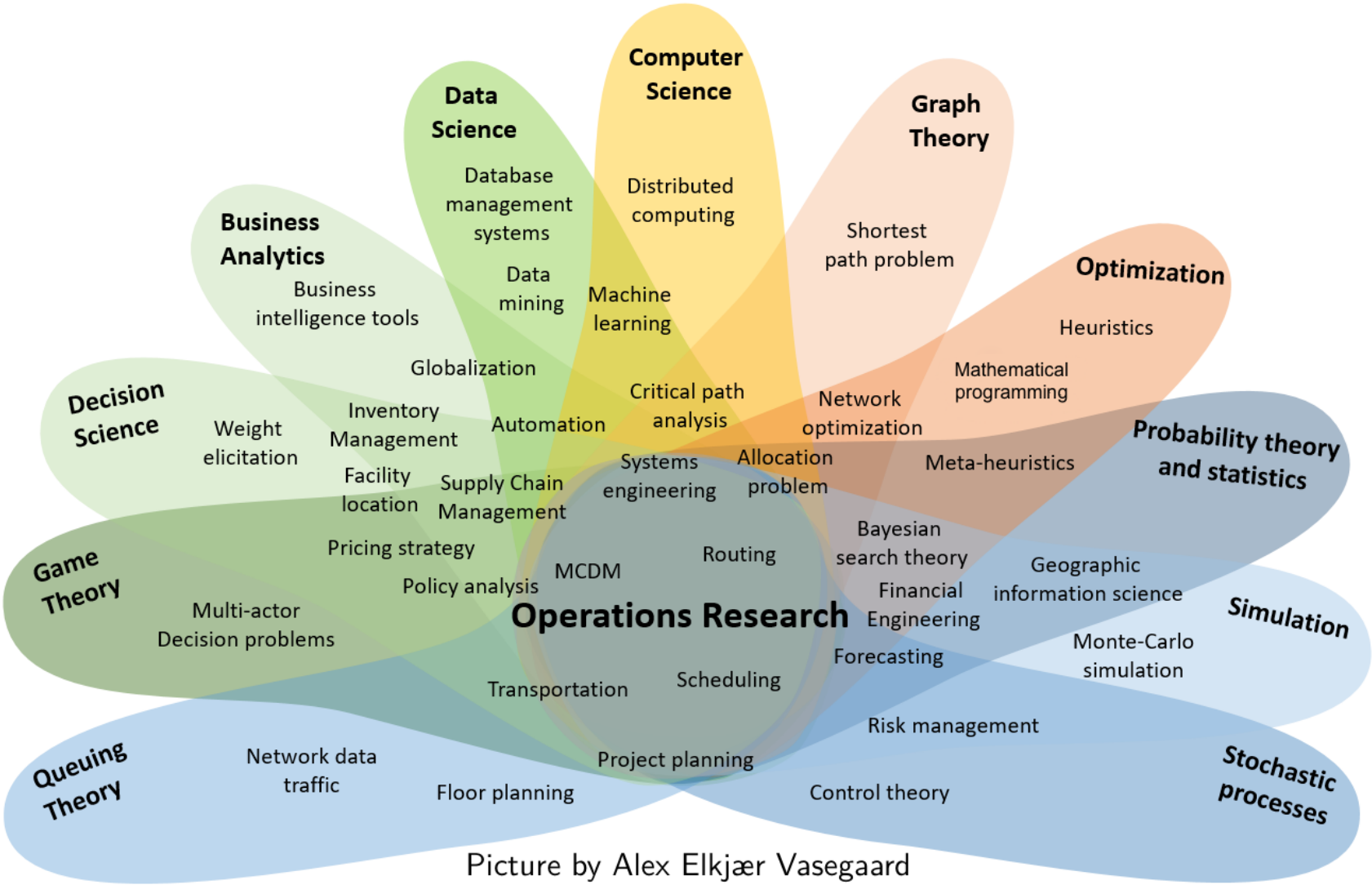
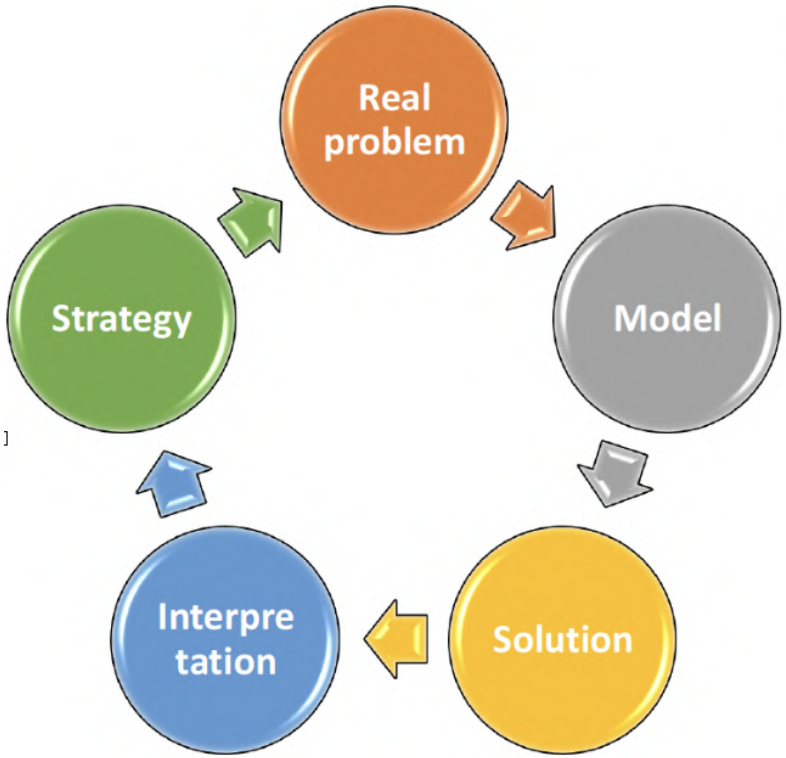
ShaKER 2025 – Santa Marta, University of Verona, December 04, 2025

What is Operations Research (OR)?



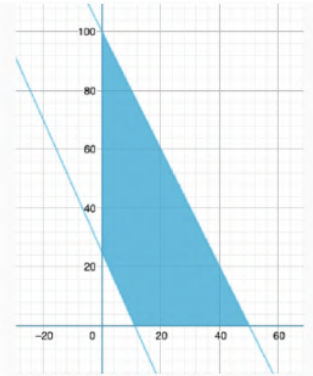
Picture by Alex Elkjær Vasegaard

OR problems and the modeling process



Problem description

A farm must determine how many hectares of land they owe be dedicated to lettuce and tomato production. By cultivating one hectare of land, the company estimated that 20 quintals of lettuce and 30 quintals of tomatoes can be produced annually. To complete the crops, one laborer has to be assigned to each hectare planted with lettuce and two laborers have to be assigned to each hectare planted with tomatoes. In order to have enough manpower for other crops, the company does not want to use more than 100 workers. Also, the company sells every kilogram of lettuce and every kilogram of tomatoes for 1 euro and 1.5 euro, respectively. Moreover, it wants to ensure an annual profit of at least 50,000 euro from the sale of these two products.



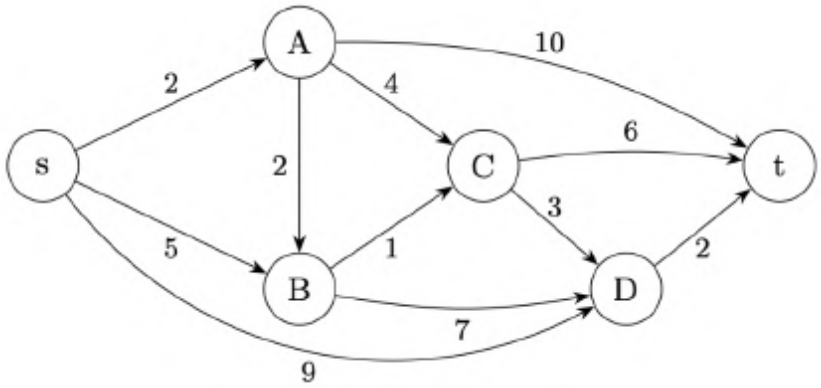
$$\min \quad x_P + x_L$$

$$2x_P + x_L \leq 100$$

$$4500x_P + 2000x_L \geq 50000$$

$$x_P, x_L \geq 0$$

How to solve OR problems



Solvers

$$\min \sum_{(i,j)\in A} c_{ij}x_{ij}$$

$$\text{s.t.} \sum_{(s,j)\in A} x_{sj} = 1$$

$$\sum_{(i,d)\in A} x_{id} = 1$$

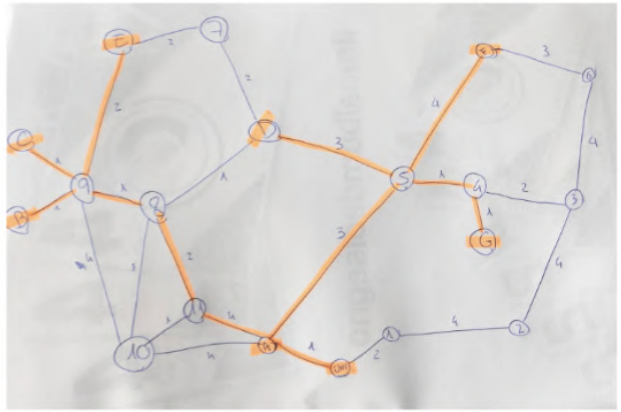
$$\sum_{(i,j)\in A} x_{ij} = \sum_{(j,k)\in A} x_{jk} \qquad j \in V \setminus \{s,d\}$$

$$x_{ij} \in \{0,1\} \qquad (i,j) \in A$$

Exact and heuristic approaches

```
1  function Dijkstra(Graph, source):
2
3      for each vertex v in Graph.Vertices:
4          dist[v] ← INFINITY
5          prev[v] ← UNDEFINED
6          add v to Q
7      dist[source] ← 0
8
9      while Q is not empty:
10         u ← vertex in Q with minimum dist[u]
11         Q.remove(u)
12
13         for each arc (u, v) in Q:
14             alt ← dist[u] + Graph.Edges(u, v)
15             if alt < dist[v]:
16                 dist[v] ← alt
17                 prev[v] ← u
18
19     return dist[], prev[]
```

Finding out about OR



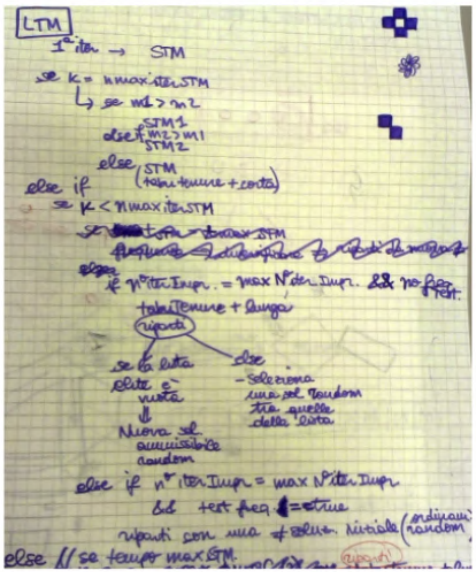
- Prova di MPL -

Problema: La ditta dolciaria TuttiGustiBenoUno sta per mettere in produzione 2 nuove tipologie di caramelle gommose. Ogni caramella, dal peso di 1 g ciascuna, dovrà essere preparata rispettando le quantità percentuali minime di ingredienti indicate nella seguente tabella.

	Tipo 1	Tipo 2
Gelatina	40	50
Zucchero	15	25
Sciroppo di frutta	30	-
Aromi naturali	5	10

Per l'acquisto delle materie prime l'azienda si rivolge ad un unico fornitore il cui listino prezzi è il seguente: 35€/kg per la gelatina, 1 €/Kg per lo zucchero, 13 €/Kg per lo sciroppo e 50 €/Kg per gli aromi. L'azienda vuole produrre per il lancio sul mercato almeno 1 030 000 caramelle di tipo 1 e almeno 1 800 000 caramelle di tipo 2. Si formuli un modello di Programmazione Lineare che aiuti l'azienda a formulare un ordine di acquisto che minimizzi i costi tenendo inoltre in considerazione che:

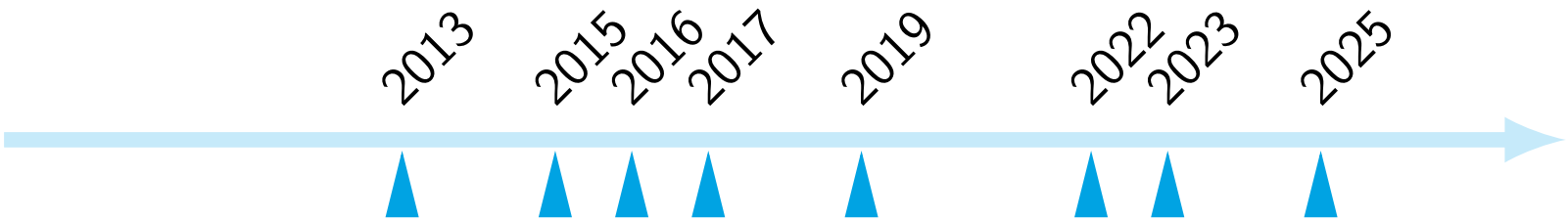
- Il peso dei prodotti finali è pari alla somma esatta del peso degli ingredienti;
- In magazzino vi sono già 10 Kg di zucchero utilizzabile.



Two fundamental courses:

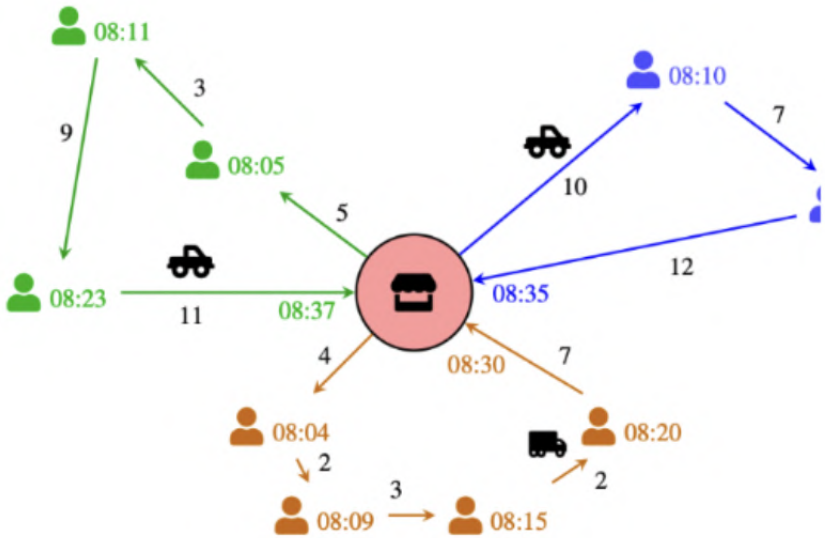
Operations Research in 2012 and *Optimization Algorithms* in 2014

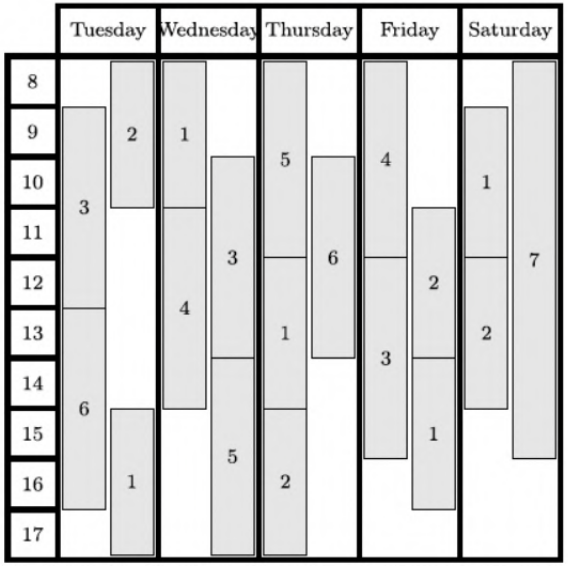
Timeline – Experiences, information



- Oct. 2013: B.Sc. in CS Engineering at UniBS
- Sep. 2015 – Jan. 2016: Visiting researcher at ULaval
- Mar. 2016: M.Sc. in CS Engineering at UniBS
- May 2019: Visiting researcher at UPEM
- Oct. – Dec. 2019: Visiting researcher at NII
- Mar. 2022: Ph.D. in Mathematics at UniTN-UniVR
- Apr. 2022 – Mar. 2023: Postdoc at UniVR
- Apr. 2023 – Mar. 2025: Postdoc/visiting at UniCal-UniPD
- Apr. 2025 – Ongoing: Postdoc at UniPD

Master thesis on routing problems





$$\min \sum_{s\in S} p_s \left[\sum_{d\in D} \sum_{(i,j)\in A_s} c_{ij} x_{ij}^{ts} + \beta \sum_{i\in N_s} z_i^s \right] \quad \text{OBJ FUN} \quad (1)$$

CONSTRAINTS

$$\text{s.t.} \sum_{w\in W_z} y_{zw} = n_z \qquad z \in Z \qquad (2)$$

$$\sum_{w\in W_z(d)} y_{zw} \le 1 \qquad z \in Z, d \in D \qquad (3)$$

$$\sum_{d\in D} z_{id}^s + z_i^s = 1 \qquad s \in S, i \in N_s \qquad (4)$$

$$\sum_{j\in V_s} x_{ji}^{ts} = z_{id}^s \qquad d \in D, s \in S, i \in N_s \qquad (5)$$

$$\sum_{j\in V_s} x_{ij}^{ts} = \sum_{j\in V_s} x_{ji}^{ts} \qquad d \in D, s \in S, i \in N_s \qquad (6)$$

$$\sum_{j\in V_s} x_{0j}^{ts} \le k_d \qquad d \in D, s \in S \qquad (7)$$

$$\sum_{i\in H} \sum_{j\in H} x_{ij}^{ts} \le |H| - \left\lceil \frac{\sum_{i\in H} q_i}{Q} \right\rceil \qquad d \in D, s \in S, H \subseteq V_s, |H| \ge 2 \qquad (8)$$

$$a_j^s \ge a_i^s + (b_i + t_{ij}) \sum_{d\in D} x_{ij}^{ts} - u_0 \left(1 - \sum_{d\in D} x_{ij}^{ts}\right) \qquad (i,j) \in A_s, s \in S \qquad (9)$$

$$a_i^s \le \sum_{w\in W_z(d)} u_w y_{zw} + u_0 (1 - z_{id}^s) \qquad d \in D, s \in S, z \in Z, i \in N_s^z \qquad (10)$$

$$a_i^s \ge \sum_{w\in W_z(d)} l_w y_{zw} - u_0 (1 - z_{id}^s) \qquad d \in D, s \in S, z \in Z, i \in N_s^z \qquad (11)$$

$$z_{id}^s + z_i^s \ge \sum_{w\in W_z(d)} y_{zw} - \sum_{k\in D} \sum_{w\in W_z(d)} y_{zw} \qquad d \in D, s \in S, z \in Z, i \in N_s^z \qquad (12)$$

VARIABLES

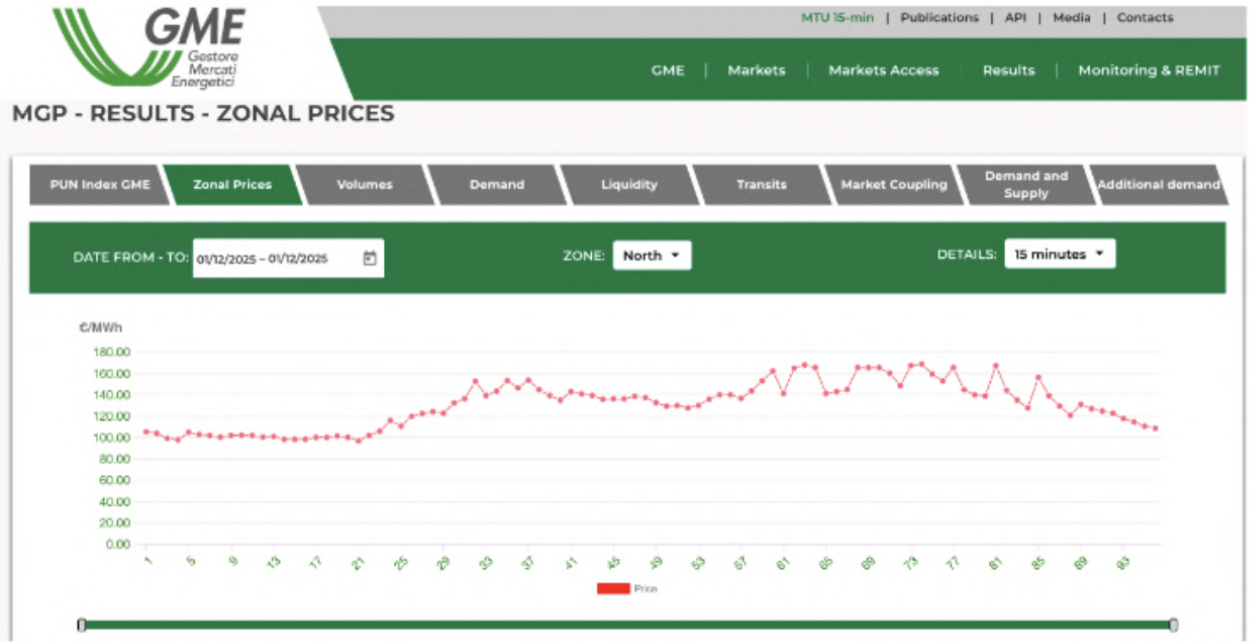
$$z_i^s, z_{id}^s \in \{0,1\} \qquad d \in D, s \in S, i \in N_s \qquad (13)$$

$$x_{ij}^{ts} \in \{0,1\} \qquad d \in D, s \in S, (i,j) \in A_s \qquad (14)$$

$$a_i^s \ge 0 \qquad s \in S, i \in N_s \qquad (15)$$

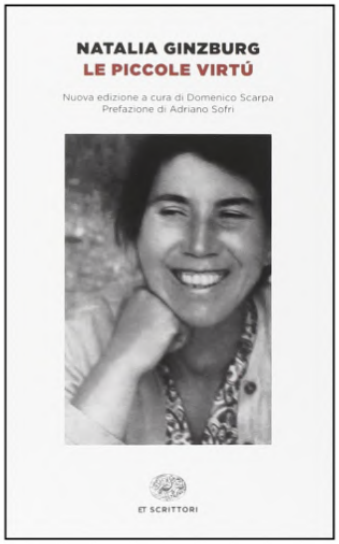
$$y_{zw} \in \{0,1\} \qquad z \in Z, w \in W_z \qquad (16)$$

Between industry and academia – A Ph.D. apprenticeship



At least, what it was supposed to be

Turning point #1 – July 2017



My vocation is to write and I have known this for a long time. I hope I won't be misunderstood; I know nothing about the value of the things I am able to write. I know that writing is my vocation. When I sit down to write I feel extraordinarily at ease, and I move in an element which, it seems to me, I know extraordinarily well; I use tools that are familiar to me and they fit snuggly in my hands.

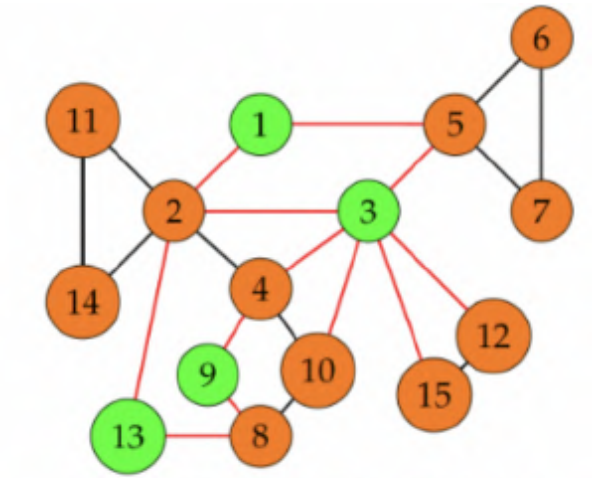
Failure makes us stronger, maybe.

Sometimes, falling helps us eliminate the unnecessary.

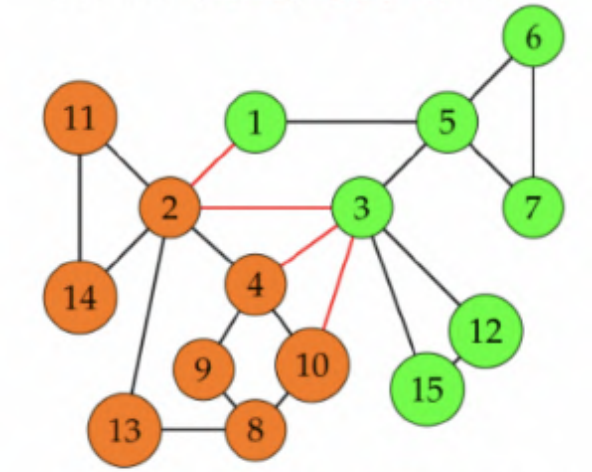
Having courage often means being strong enough to accept defeat.

My Ph.D. experience – Research on enumeration problems

Listing all *bonds* of a connected graph



(a) Given $\{S_1, \overline{S_1}\}$, $\delta_G(S_1, \overline{S_1})$ is a *cut*.



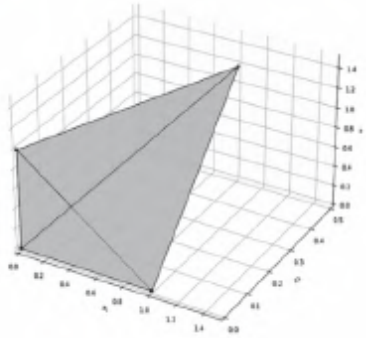
(b) Given $\{S_2, \overline{S_2}\}$, $\delta_G(S_2, \overline{S_2})$ is a *bond*.

Deciding the *duality* of two monotone Boolean functions

$$f(x) = \bigvee_{F \in \mathcal{F}} \bigwedge_{i \in F} x_i =: \mathrm{DNF}(\mathcal{F})$$

$$g(x) = \bigwedge_{G \in \mathcal{G}} \bigvee_{i \in G} x_i =: \mathrm{CNF}(\mathcal{G})$$

$$\begin{cases} x_1 + x_2 + x_3 \leq 1 \\ x_1 + 2x_2 + 2x_3 \leq 2 \\ x_1 - x_2 \leq 3 \\ x_1, x_2, x_3 \geq 0 \end{cases}$$



My Ph.D. experience – Collaborations with companies

Home > Operations Research and Data Science in Public Services > Conference paper

A Two-Phase Approach to Evaluate and Optimize an Interlibrary Loan Service: The Case Study of *Provincia di Brescia*

Conference paper | First Online: 19 August 2023

pp 17–33 | Cite this conference paper

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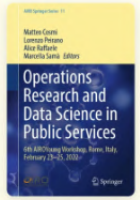
Alice Raffaele, Matteo Zavatteri, Fabio Bazzoli, Marco Gussago & Romeo Rizzi

Part of the book series: AIRO Springer Series ((AIROSS,volume 11))

Included in the following conference series: Workshop on Operation Research and Data Science in Public Services



Operations Research and Data Science in Public Services

(AIROYoung 2022)

Sections | Figures | References

Abstract

Introduction and Motivation

The Interlibrary Loan Service of Provincia di Bresc

Related Work

UNIVERSITÀ di VERONA

Un approccio innovativo per l'etichettatura degli intervalli di riposo

Dott. Enrico Paganin
Dott. Alice Raffaele
Prof. Romeo Rizzi, PhD
Ing. Andrea Saviozzi

Maggio 2020

My Ph.D. experience – Teaching various OR courses


Hands-on tutorial on Linear Programming and Integer Linear Programming modeling

Introduction to AMPL and GMPL

Romeo Rizzi, Alice Raffaele

University of Verona

romeo.rizzi@univr.it, alice.raffaele@unitn.it

November 20, 2018

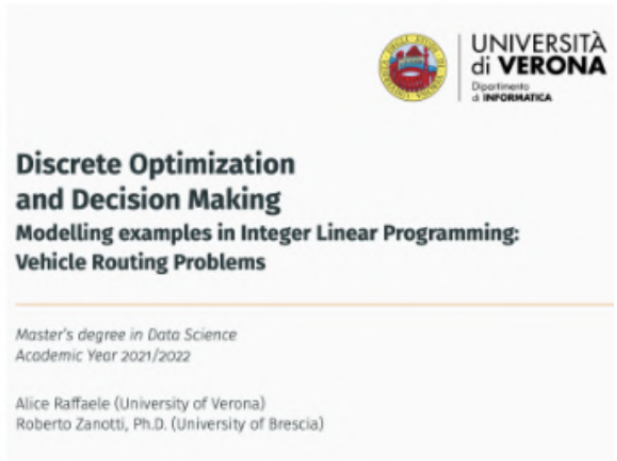

Tutoraggio Ricerca Operativa 2019/2020
0. Introduzione

Alice Raffaele, Romeo Rizzi

Università degli Studi di Verona

31 marzo 2020

Raffaele, Rizzi (UniVR) | Tutoraggio RO 2019/2020 | 9 giugno 2020 | 1 / 12

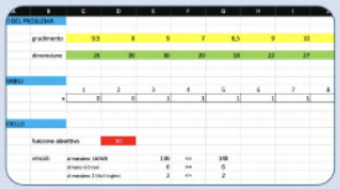

UNIVERSITÀ di VERONA
Dipartimento di INFORMATICA

Discrete Optimization and Decision Making

Modelling examples in Integer Linear Programming: Vehicle Routing Problems

Master's degree in Data Science
Academic Year 2021/2022

Alice Raffaele (University of Verona)
Roberto Zanotti, Ph.D. (University of Brescia)



ROAR I – Grade 10
(March – May 2021)

Mathematical modelling

Linear, integer, and mixed-integer linear programming

Digital technologies: GeoGebra, Microsoft Excel add-in Solver, Mentimeter

ROAR II – Grade 11
(January – April 2022)

Graph theory and network applications (Minimum spanning tree, Shortest path, Travelling Salesman problems)

Heuristics and pseudocodes

Digital technologies: + Kahoot

ROAR III – Grade 12
(October 2022 – January 2023)

Implementation of mathematical models in Python

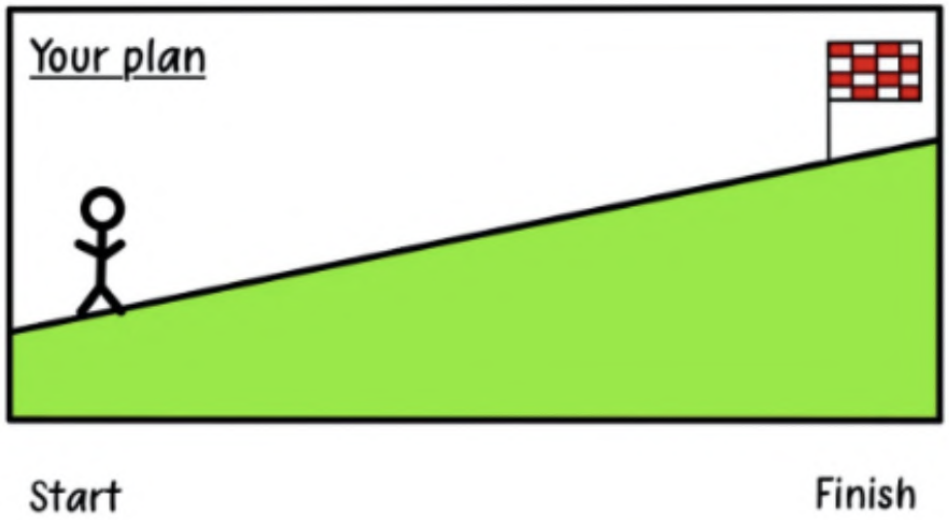
Resolution by means of PuLP

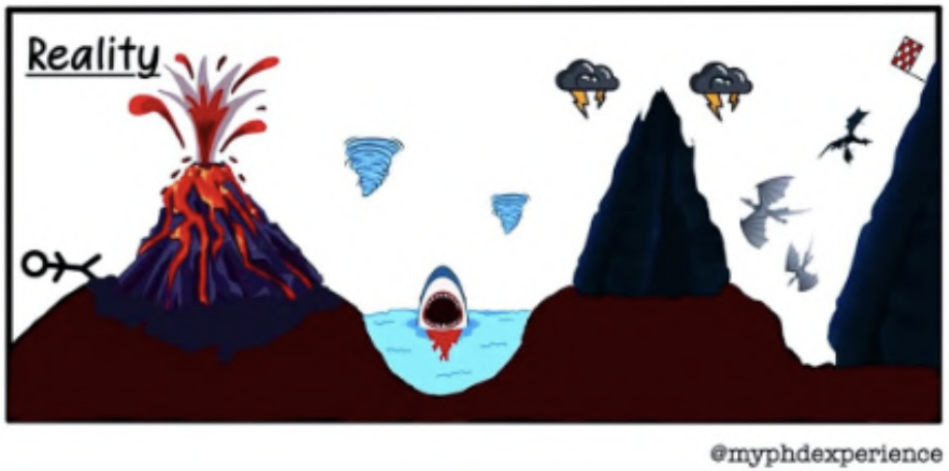
Digital technologies: + Spyder

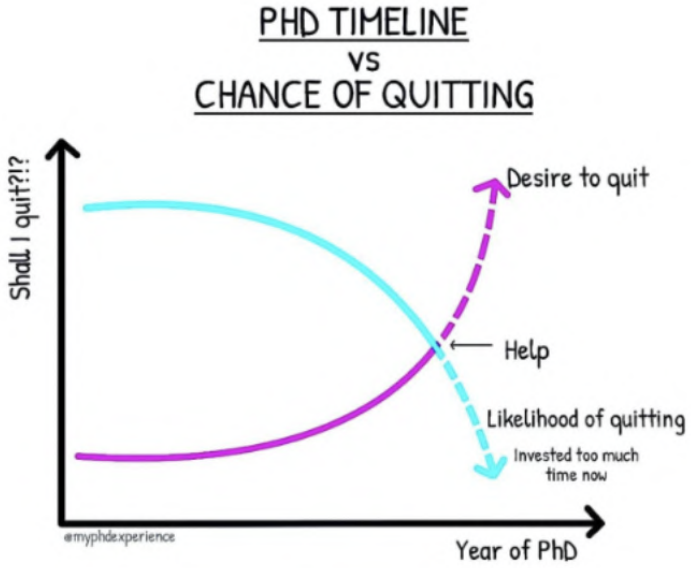
Turning point #2 – October 2018

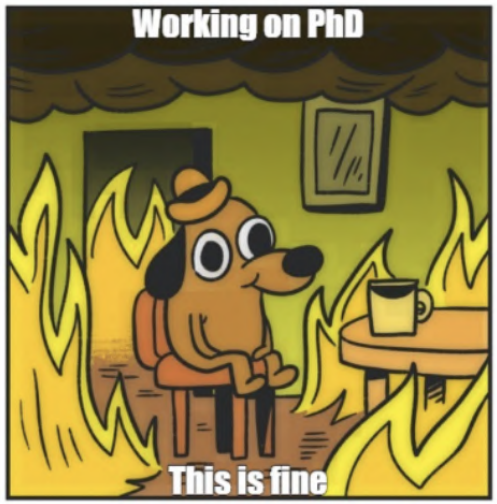
We all know it: a single hour of class can change a life, shape destiny in a different direction, forever seal what was only faintly outlined. We've all experienced what a single hour of class can be: visiting another place, another world, being transported, catapulted into another world, encountering the unexpected, the marvelous, the unprecedented.

My Ph.D. experience – The dark side

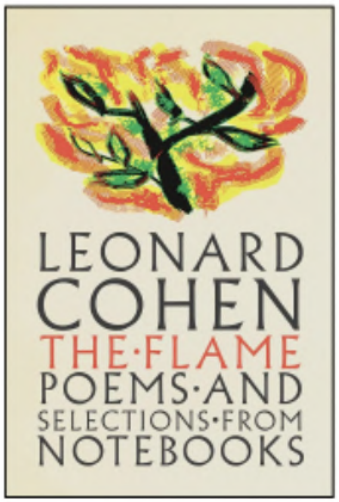








Turning point #3 – October 2019 and August 2020

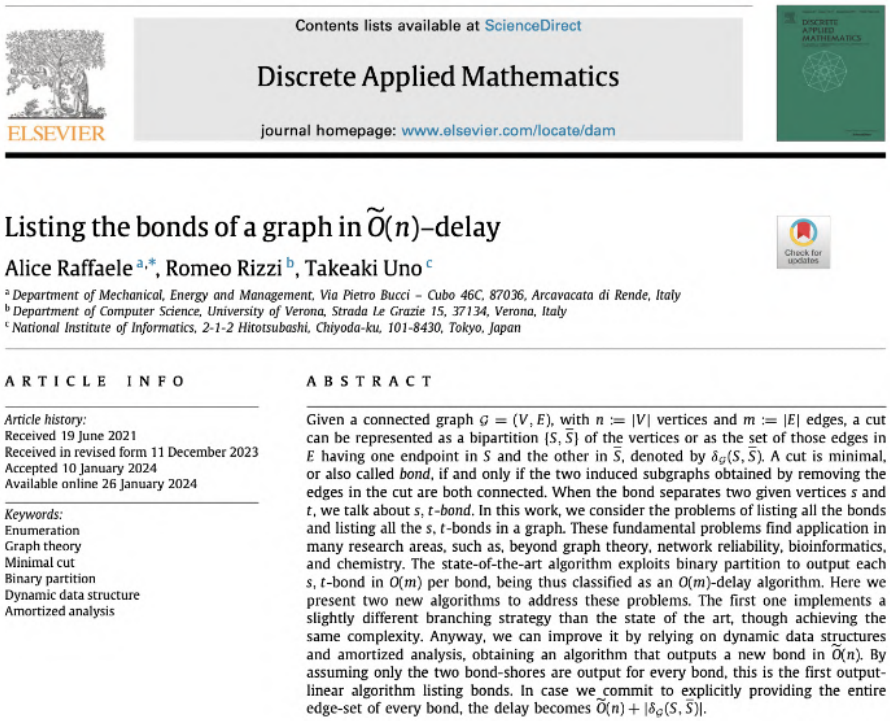


There is a crack, a crack in everything:
that's how the light gets in.

Just one recommendation: think. [...]
On the door. The narrow door. The only door.
The one that leads to the corridor you wish to enter.
Chopin chose the door of music,
Callas the door of singing.
Which is your door?

My Ph.D. experience – Talking

My Ph.D. experience – Research outputs



Contents lists available at ScienceDirect

Discrete Applied Mathematics

journal homepage: www.elsevier.com/locate/dam

Listing the bonds of a graph in $\widetilde{O}(n)$–delay

Alice Raffaele [a,*], Romeo Rizzi [b], Takeaki Uno [c]

[a] Department of Mechanical, Energy and Management, Via Pietro Bucci – Cubo 46C, 87036, Arcavacata di Rende, Italy
[b] Department of Computer Science, University of Verona, Strada Le Grazie 15, 37134, Verona, Italy
[c] National Institute of Informatics, 2-1-2 Hitotsubashi, Chiyoda-ku, 101-8430, Tokyo, Japan

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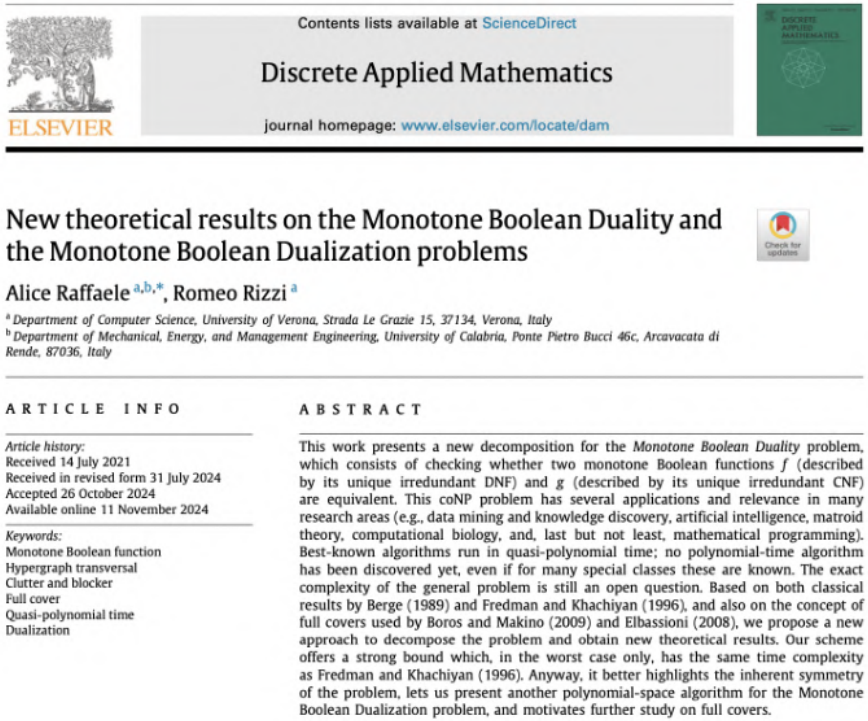
Keywords:
Enumeration
Graph theory
Minimal cut
Binary partition
Dynamic data structure
Amortized analysis

ABSTRACT

Given a connected graph $G = (V, E)$, with $n := |V|$ vertices and $m := |E|$ edges, a cut can be represented as a bipartition $\{S, \overline{S}\}$ of the vertices or as the set of those edges in E having one endpoint in S and the other in $\overline{S}$, denoted by $\delta_G(S, \overline{S})$. A cut is minimal, or also called *bond*, if and only if the two induced subgraphs obtained by removing the edges in the cut are both connected. When the bond separates two given vertices s and t, we talk about s, t-*bond*. In this work, we consider the problems of listing all the bonds and listing all the s, t-bonds in a graph. These fundamental problems find application in many research areas, such as, beyond graph theory, network reliability, bioinformatics, and chemistry. The state-of-the-art algorithm exploits binary partition to output each s, t-bond in $O(m)$ per bond, being thus classified as an $O(m)$-delay algorithm. Here we present two new algorithms to address these problems. The first one implements a slightly different branching strategy than the state of the art, though achieving the same complexity. Anyway, we can improve it by relying on dynamic data structures and amortized analysis, obtaining an algorithm that outputs a new bond in $\widetilde{O}(n)$. By assuming only the two bond-shores are output for every bond, this is the first output-linear algorithm listing bonds. In case we commit to explicitly providing the entire edge-set of every bond, the delay becomes $\widetilde{O}(n) + |\delta_G(S, \overline{S})|$.



Contents lists available at ScienceDirect

Discrete Applied Mathematics

journal homepage: www.elsevier.com/locate/dam

New theoretical results on the Monotone Boolean Duality and the Monotone Boolean Dualization problems

Alice Raffaele [a,b,*], Romeo Rizzi [a]

[a] Department of Computer Science, University of Verona, Strada Le Grazie 15, 37134, Verona, Italy
[b] Department of Mechanical, Energy, and Management Engineering, University of Calabria, Ponte Pietro Bucci 46c, Arcavacata di Rende, 87036, Italy

ARTICLE INFO

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Keywords:
Monotone Boolean function
Hypergraph transversal
Clutter and blocker
Full cover
Quasi-polynomial time
Dualization

ABSTRACT

This work presents a new decomposition for the *Monotone Boolean Duality* problem, which consists of checking whether two monotone Boolean functions f (described by its unique irredundant DNF) and g (described by its unique irredundant CNF) are equivalent. This coNP problem has several applications and relevance in many research areas (e.g., data mining and knowledge discovery, artificial intelligence, matroid theory, computational biology, and, last but not least, mathematical programming). Best-known algorithms run in quasi-polynomial time; no polynomial-time algorithm has been discovered yet, even if for many special classes these are known. The exact complexity of the general problem is still an open question. Based on both classical results by Berge (1989) and Fredman and Khachiyan (1996), and also on the concept of full covers used by Boros and Makino (2009) and Elbassioni (2008), we propose a new approach to decompose the problem and obtain new theoretical results. Our scheme offers a strong bound which, in the worst case only, has the same time complexity as Fredman and Khachiyan (1996). Anyway, it better highlights the inherent symmetry of the problem, lets us present another polynomial-space algorithm for the Monotone Boolean Dualization problem, and motivates further study on full covers.

On average, they took three years to be published

My Ph.D. experience – Service and networking

AIRO YOUNG

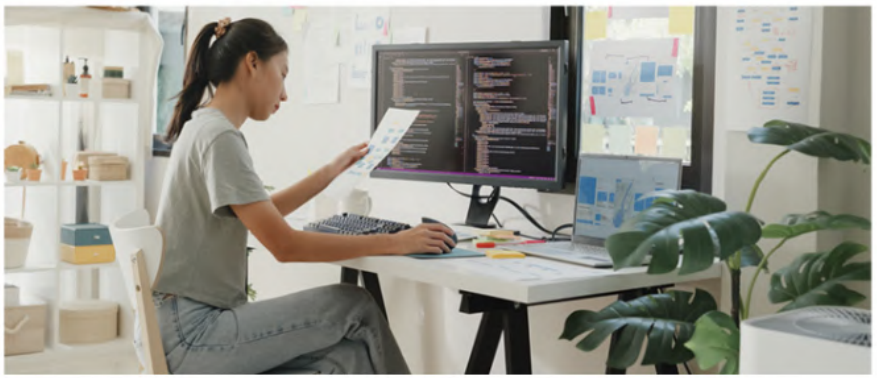
ORMS TODAY informs membership magazine

NEWS FEATURES PODCASTS DEPARTMENTS ISSUES

If Dantzig Had a Sister

By Alice Raffaele, Anand Subramanian

SHARE: PRINT ARTICLE: https://doi.org/10.1287/orms.2024.01.06



February 11 is the International Day of Women and Girls in Science. We exploit some quotes from Virginia Woolf's "A Room of One's Own" [1] to discuss why there is still the need for such days as well as the challenges faced by women in operations research (O.R.) past and present.

MaddMaths!

MAtematica Divulgazione Didattica

Turning point #4 – September 2022

Here's the paradox: though much of a mathematician's work is solitary, even lonely, the most important aspect of your research is not the field you choose or the problems you embark on but how you deal with the people around you. When you set out to earn a PhD, you do not do so alone.

Research network

Ongoing research projects

Last-mile Delivery

Mixed Integer Linear Programming

Fleet Sizing

Robust Optimization & Appr. Dyn. Program.

Urban Air Mobility

Exact methods: Branch &Cut, Branch&Price

Warehouse Managem.

Meta-heuristics & Matheuristics

Uncertain Knapsack

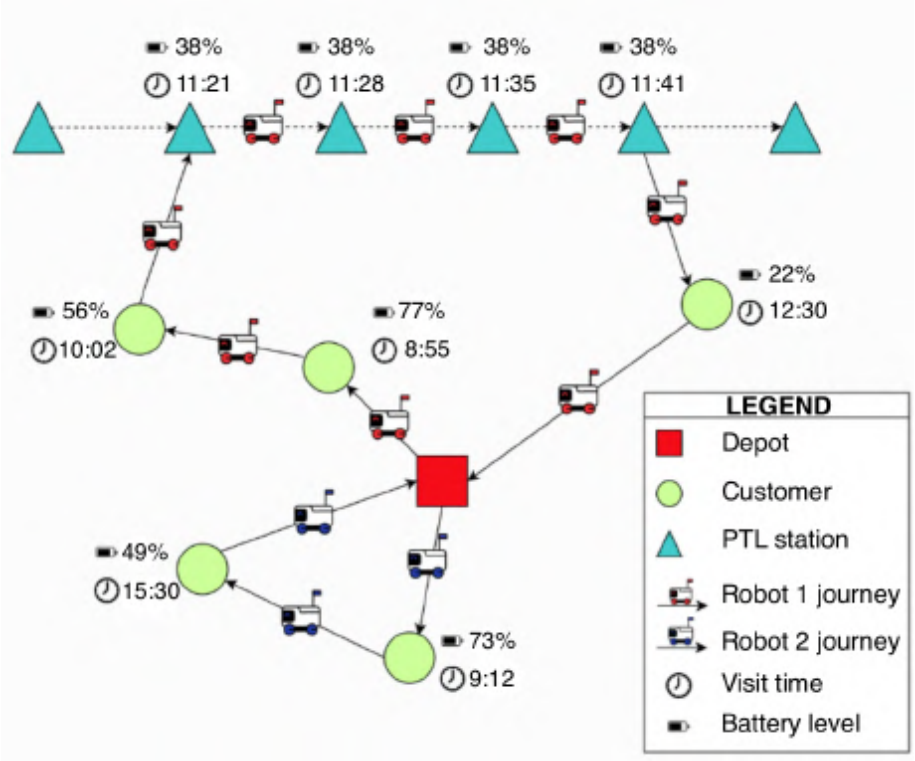
Exact methods: Dyn. Program.

OR Teaching

Active learning & Constructivism

Last-mile Delivery with Autonomous Robots and Public Transport Lines – Problem Definition

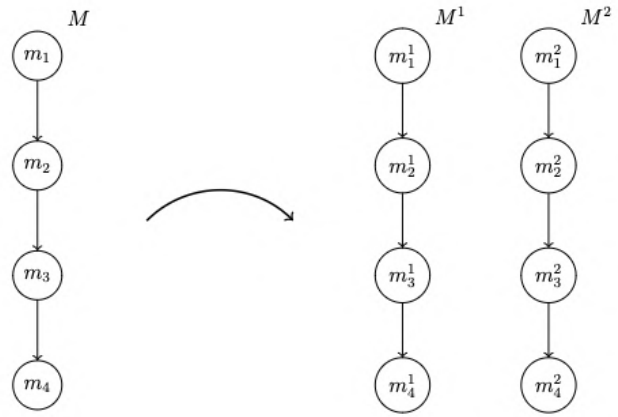
Joint work with Antonino A. Carè (Politecnico di Milano), Roberto Roberti (University of Padova), Sara Stoia (University of Calabria), and Yanlu Zhao (Durham University)



- Single depot, homogeneous robots, customers, and **metro stations**
- Payload-dependent energy consumption and battery swap
- **Goal**: minimize total cost to serve customers

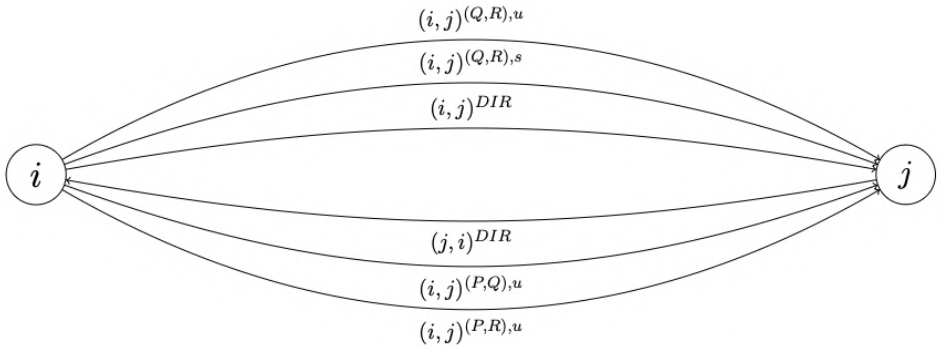
Last-mile Delivery with Autonomous Robots and Public Transport Lines – Methodology and Results

1 Shift network:

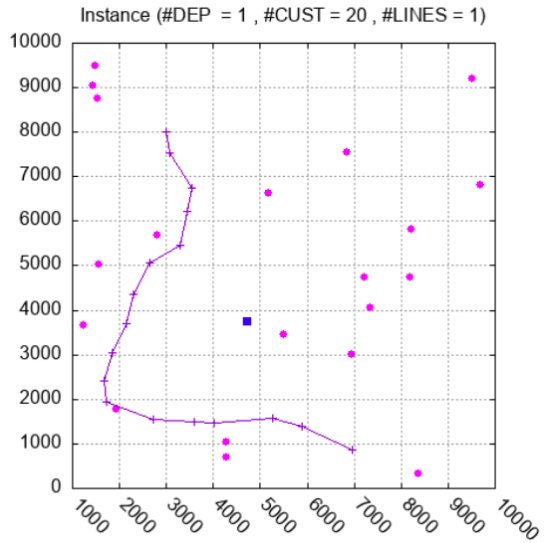


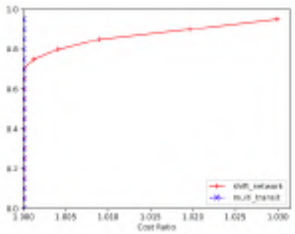
$\rightarrow$ Traditional x_{ij}

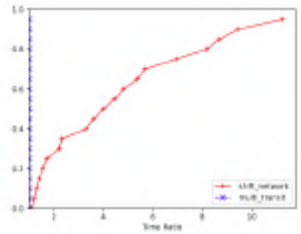
2 Multi-transit network:



$\rightarrow x_{ij}^{\text{DIR}}$ and $x_{ij}^{\text{PR},s}$ + dominance rule





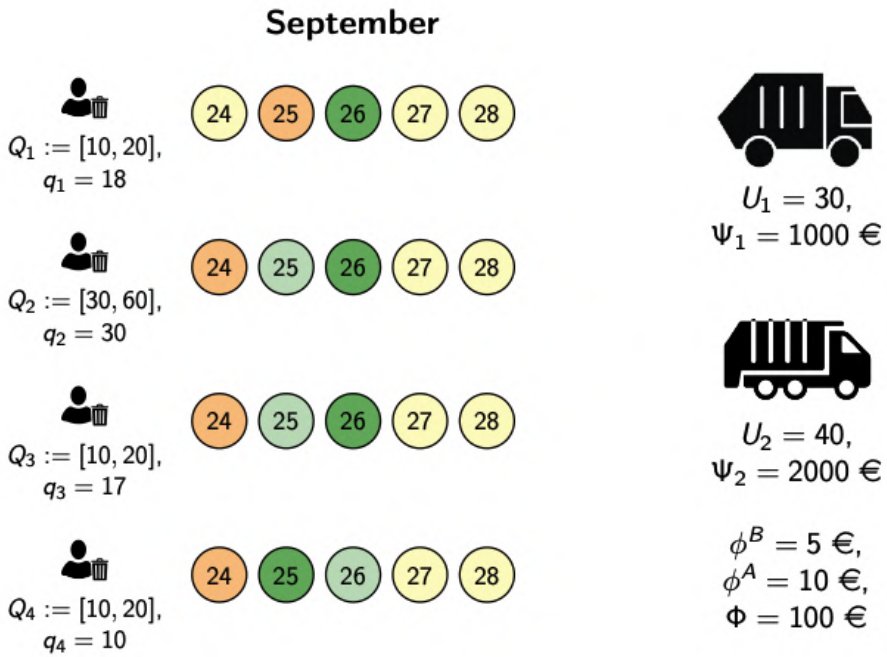


- Realistic instances with 20, 40, or 60 customers and 2, 4, or 8 robots
- Sensitivity analyses

Robust Policies for a Multi-Period Fleet Sizing Problem under Demand Uncertainty – Problem Definition

Joint work with Demetrio Laganà (University of Calabria) and Roberto Roberti (University of Padova)

- Stochastic-dynamic setting over multiple days
- Customers with **uncertain demand in known intervals**, day windows, and penalties
- Vehicles with $\neq$ capacities and fixed costs
- **Goal**: minimize total costs due to fleet sizing, staffing, and customer inconvenience



Optimal solution (value = 2205 €) – Employ vehicle 2 to serve:

- Customers 1 and 3 on 09/26
- Customers 2 and 4 on 09/25

Robust Policies for a Multi-Period Fleet Sizing Problem under Demand Uncertainty – Methodology

Lower bounds:

1. EP – Deterministic ex-post:
   - Variables for service, usage, and number of vehicles of each type each day
   - Constraints on service, capacity, vehicle availability
2. QMIN – Pending orders in the objective function and uncertain orders with minimum demand in constraints

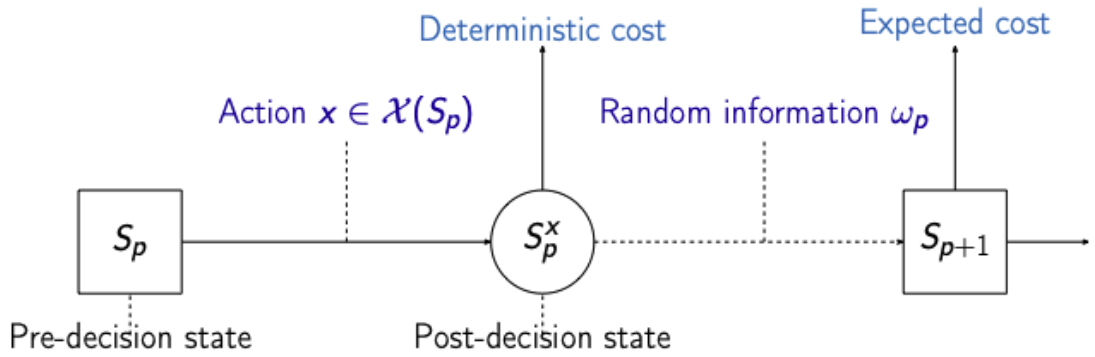
Upper bounds:

1. WC – Worst case (maximum demand)
2. RO – Robust counterpart: **budget of uncertainty** and cardinality-constrained uncertainty set

$$\sum_{i \in N_0} q'_i a_{ikp} + \sum_{i \in \tilde{N}} \underline{q}_i a_{ikp} + \max_{\substack{I \subseteq \tilde{N} \\ |I| \leq \Gamma}} \sum_{i \in S} (\overline{q}_i - \underline{q}_i) a_{ikp} \leq R_k u_{kp} \quad k \in K, p \in P \setminus \{0\}$$

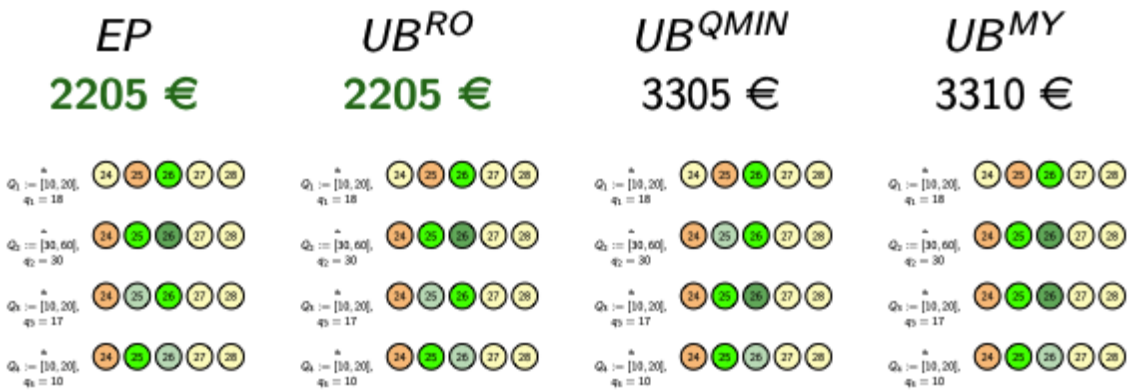
Linearization through dualization or recursive equations

Robust Policies for a Multi-Period Fleet Sizing Problem under Demand Uncertainty – Methodology and Results



Definition of **Approximate Dynamic Programming** policies applied in a folding-horizon fashion to provide **upper bounds**

$$\text{QMIN} \leq \text{EP} \leq \text{RO}, \textbf{UB} \leq \text{WC}$$

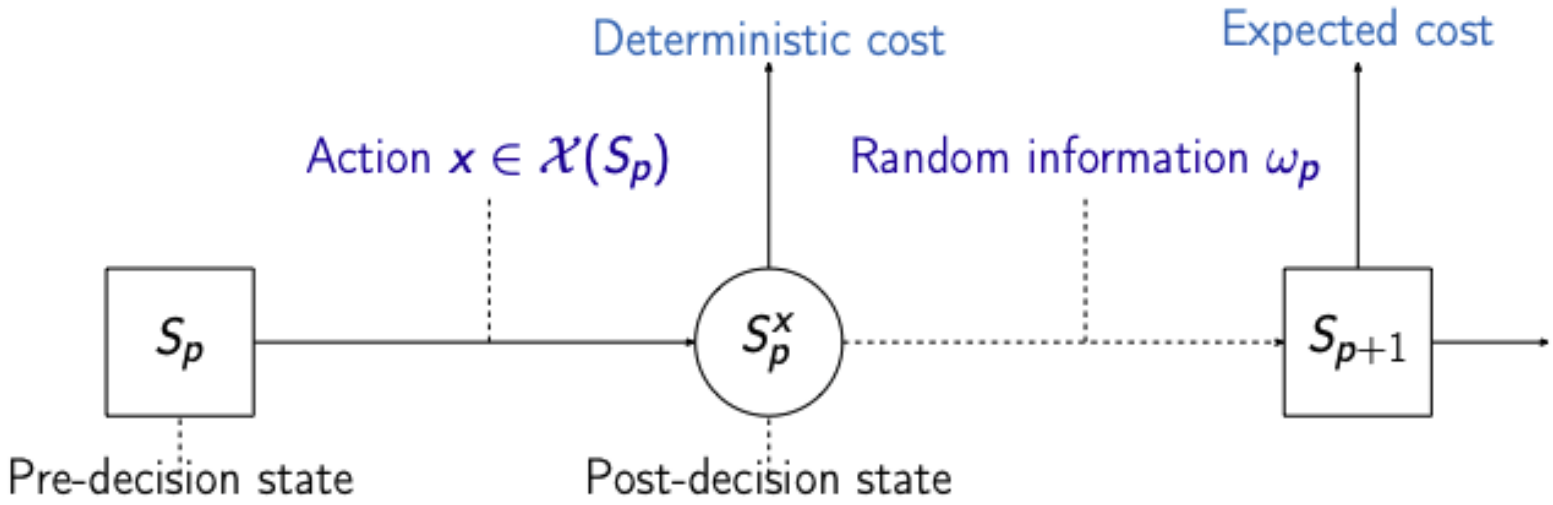


Conclusions – Eight memos for the next researchers

Who are we, who is each one of us, if not a combinatoria of experiences, information, books we have read, things imagined? Each life is an encyclopedia, a library, an inventory of objects, a series of styles, and everything can be constantly shuffled and reordered in every way conceivable.

1. *Nana korobi, ya oki*
2. Nonlinearity is perfectly fine
3. Experience something before judging it
4. When life gives you lemon, make your best lemonade, and plant lemon seeds
5. Talk to as many people as you can
6. Aim to one primary topic and many secundary others
7. Keep an eye on the present goals and the other ahead
8. Answer the question: Why am I pursuing a research career?

Conclusions – A path of one's own



- **Problem**: Pursuing a research career
- **Model**: Stochastic-dynamic problem with personal decision variables and constraints
- **Goals**:
  - Learning one's own policies
  - Balancing these with all the rest you cannot control

Events – 10th AIROYoung Workshop and Ph.D. School

Website: `https://ayw2026.github.io`
Abstract submission deadline: December 14, 2025
Paper submission deadline: February 14, 2026

Events – Creative Math. Sciences Communication 2026

CMSC 2026 Important Dates Call Schedule Speakers Sponsors Committees Venue Events

Creative Mathematical Sciences Communication 2026

Padova, October 12–15, 2026

Email

Creative Mathematical Sciences Communication 2026

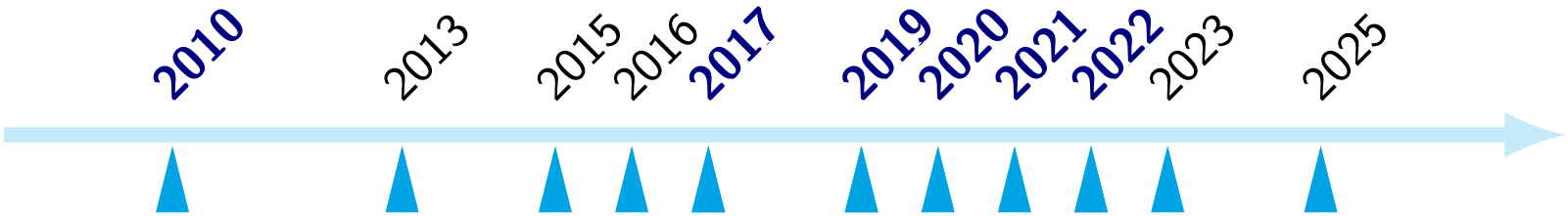
We are pleased to invite you to the 8th edition of the international conference **Creative Mathematical Sciences Communication**, which will be held in Padova, Italy, from October 12 to October 15, 2026.

The CMSC conference series was initiated in 2013, recognizing that the competencies required for the increasingly computerized world are essentially mathematical. The CMSC aims to explore new ways of communicating the rich mathematical ideas fundamental for computer science. The science of computer science is mathematics. "*Turn off the machines and turn on your mathematical thinking.*" For an example, see "Games That Cannot Go on Forever! Active Participation in Research Is the Main Issue for Kids" by Michael R. Fellows, Frances A. Rosamond. CMSC 2024 Proceedings, Trier, Germany.

There's a need and an appetite by mathematicians and computer scientists for outreach and engagement at all levels. Bringing together researchers, educators and communicators, the conference hopes to explore new ways of mathematical-computational thinking and lay the groundwork for innovative teaching and research engagement approaches and methods at all education levels.

Website: `https://cmsc2026.github.io`
Paper/poster submission deadline: TBA

Timeline – Books



- May 2010: Virginia Woolf, "A room of one's own" – Independence and equality
- June 2017: Natalia Ginzburg, "The little virtues" – Vocation and relationships
- Jul. 2017: Pirkko Vainio, "For those who want to fly" – Falling and standing up
- Oct. 2018: Massimo Recalcati, "The class hour" – Impact of teaching
- Dec. 2019: Leonard Cohen, "The flame" – Resignation, cracks, and gratitude
- May 2020: Italo Calvino, "Invisible cities" – Research
- Aug. 2020: É-E. Schmitt, "M.me Pylinska and Chopin's secret" – Finding one's door
- Dec. 2020: Kobi Yamada, "Maybe" – Personal qualities
- Nov. 2021: Italo Calvino, "Six memos for the next millennium" – Pure principles
- Dec. 2021: Chandra Livia Candiani, "Questo immenso non sapere" – Role of errors
- Sep. 2022: Ian Steward, "Letters to a young mathematician" – Research community

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